## Green Level

1. (5 credits) First three terms of the arithmetic progression are arranged in a line. The same numbers are replaced with the same letters, different numbers are replaced with different letters. The result of the replacement is A BC AAA. Which digit was replaced by the letter C?
(Answer: 6.)

**Solution**. The only possible option for AAA is 111. Therefore A=1, BC=(1+111)/2=56.

2. (6 credits) Professor Zero jetted off to the conference to Toronto from Moscow. His aircraft left Moscow at 06.35PM and arrived to Toronto on the next day at 10:20AM (the time of departure and the time of arrival are local). When it was 2 o'clock in the morning in Moscow, it was 7 o'clock in the previous evening in Toronto. How long did the professor's flight last?
(Answer: 8 hours 45 minutes.)

**Solution**. The time difference between Toronto and Moscow is 2+24-19=7 hours. Thus, the professor's journey began at 01:35AM Toronto time. Therefore its duration was 15 minutes less than 9 hours, i.e. 8 hours 45 minutes.

3. (6 credits) Six members of the team were solving 9 problems. 2 hours later they found that each member solved 2 problems, and each problem solved by them was solved by three members. How many problems were not solved by the team?
(Answer: 5.)

**Solution**. All members solved 6x2=12 problems, and each problem was solved three times. Therefore, 4 different problems were solved, and 5 different problems were not solved.

4. (8 credits) Five consecutive terms of the arithmetic progression with difference 6 are prime numbers (i.e. they have no positive divisors other than 1 and itself). What is the smallest term of this progression?
(Answer: 5.)

**Solution**. One of the terms of such progression can be definitely divided by 5, therefore it is a prime number only in case it is 5.

5. (10 credits) Trading in shares is opened every day at 10.00AM on the Stock Exchange in New Vasyuki. On the morning of January 1, 201N, the cost of each share of the companies "Vasya Inc." and "Petya and Co" was one ruble and two rubles respectively. On December 31 of the same year, post-trading prices for the shares became the same again. Lyosha found that the prices for the shares of those companies were always different, they changed every day, but the cost of each share was either one ruble or two rubles in any case. How many days were in February that year?
(Answer: 29.)

**Solution**. If the cost of a share on January 1 was 1 ruble, then post-trading price on January 1 became 2 rubles, on January 2 its post-trading cost was one ruble, and on January 3 its post-trading cost was 1 ruble again. This means that a share costs 2 rubles each odd day of the year, and December 31 should be the 366th day of the year. Therefore, this is a leap year.

6. (10 credits) 37 inhabitants of the island of knights and liars sat in the room. At some point, one offended inhabitant left the room. One of the remaining inhabitants followed him with his eyes and remarked, "The one who left the room is a liar!" Then he got up and left the room, too. The second one said, "Both those who left the room are liars!" and left the room as well. Then the remaining inhabitants said one after another, "All those who left the room are liars!" and left the room. Then

the last inhabitant remaining in the room stated sadly, "Yes, all those who left the room are liars." How many liars were in the room originally? (Liars always lie, and knights always tell the truth)
(Answer: 36.)

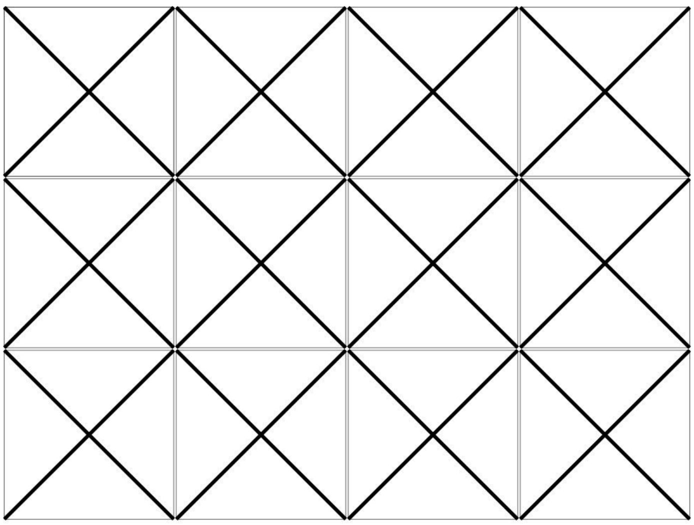
**Solution**. If at least someone told the truth, all those who spoke before him were liars, and he was a knight. Therefore, all those who spoke after him (provided that he is not the last one) were liars as well. In this case, there were 36 liars and 1 knight. And if no one told the truth, then the first one who left the room was not a liar, and this means that he was a knight, and all the others lied, and in this case there were 36 liars again.

7. (10 credits) In McDonald's, the middle set of 3 hamburgers, 5 milk shakes and 1 pack of French fries costs 235 rubles, and the large set of 5 hamburgers, 9 milk shakes and 1 pack of French fries costs 395 rubles. How much does the small set of 2 hamburgers, 2 milk shakes and 2 packs of French fries cost there if all prices are determined correctly and without discounts?
(Answer: 150 rubles. )

**Solution**. Let G be the price of a hamburger, M be the price of a cocktail, and K be the price of a pack of French fries. It follows from the equations $3G+5M+K=235$ and $5G+9M+K=395$ that $G+M+K=2(3G+5M+K)-(5G+9M+K)=470-395=75$, therefore $2(G+M+K)=150$.

8. (13 credits) Masha wrote four consecutive natural numbers on the board. Lyosha divided each of them by 10 and erased all digits after the decimal point. The sum of the new numbers written on the board was 2017. Which numbers were originally written by Masha?
(Answer: 5047, 5048, 5049, 5050.)

**Solution**. Since the remainder of dividing 2017 by 4 is 1, there are three identical numbers and one number that is larger by one, i.e. (2017-1) / 4 and (2017-1) / 4+1, that is 504 and 505. Therefore the original numbers were 5050 and 5049, 5048, 5047.



9. (15 credits) Winnie-the-Pooh had a 3×4 grid rectangle. He folded it several times along the grid lines and got a 1x1 square. Then Winnie-the-Pooh cut the received square package along both diagonals. How many parts did Winnie-the-Pooh get from the original rectangle?
(Answer: 31.)

**Solution**. The number of the parts is exactly equal to the number of the sides of the one cell long grid, and there are 16 such horizontal sides and 15 such vertical sides in total. See fig. (cuts are shown as bold lines).

10. (17 credits) Petya tries to find grid rectangles, diagonals of which cross exactly 17 squares. He has already found 1×17 and 17×17 rectangles. How many more such rectangles can he find? (a×b and b×a rectangles are assumed to be identical)
(Answer: two: 5×13 and 7×11.)

**Solution**. The diagonal of the a×b rectangle intersects all a-1 horizontal lines and b-1 vertical lines. If the numbers a and b do not have common divisors, all these lines are intersected at different points, and there are (a-1)+(b-1) intersection points on the diagonal, and it is divided into a+b-1 line segments, i.e. the diagonal intersects a+b-1 cells. This gives us the equation a+b-1=17, i.e. a+b=18 (provided that there are no common divisors). To enable a and b (the sum of which is 18) not to have common divisors, each term must be odd and not divisible by 3. This happens for pairs (1,17), (5,13) and (7,11). If a and b have common divisors, then some points of intersection of the diagonal with the horizontals and verticals coincide, and the number of such points is equal to LCD(a,b) (LCD means 'least common divisor'). Reasoning similarly to the main case, we may conclude that this diagonal intersects a+b-LCD(a,b) cells. The equality a+b-LCD(a,b)=17 may be valid for the case LCD(a,b)> 1 only if LCD(a,b)=17, because the left side is divisible by LCD(a,b). For this case we have a=17a', b=17b', a+b=34, a'+ b'=2, therefore a'= b'=1. Thus, the only option a=b=17 already found by Petya may be added here.